

1. A. $3x^4\psi, -4\psi x^4$
 $4x\omega^2, 6\omega^2x$
 $-4x^3\psi^4, \psi^4x^3$
 $5x^3\psi^4k, -k\psi^4x^3$

B. α) $\underline{3x^4} - \underline{2x^3} + \underline{5x^2} - 4x + \underline{3x^2} + \underline{x^4} = 3x^4 + x^4 - 2x^3 + 5x^2 + 3x^2 - 4x$
 $= 4x^4 - 2x^3 + 8x^2 - 4x$

β) $\underline{2x^2\psi} + \underline{3x\psi^2} + \underline{5\psi x} - \underline{3x^2\psi^2} + \underline{2\psi^2x} + \underline{5x^2\psi} - \underline{4x^2\psi} - \underline{x\psi} =$
 $= -3x^2\psi^2 + 7x^2\psi + 5x\psi^2 + 4x\psi$

Γ. α) 1) $(2x^2y)(-5xy^2) = -10x^3y^3$

2) $4x(x^3 - 3x^2 + 4x - 2) = 4x^4 - 12x^3 + 16x^2 - 8x$

3) $(x^2 + x + 3)(x + 2) = x^3 + x^2 + 2x + x^2 - 3x - 6 = x^3 + 2x^2 - x - 6$

4) $-(2x+1)(4x+2)(x-1) = -(2x+1)(4x^2 - 4x + 2x - 2)$
 $= -(8x^3 - 8x^2 + 4x^2 - 4x + 4x^2 - 4x + 2x - 2)$
 $= -8x^3 + 8x^2 - 4x^2 + 4x - 4x^2 + 4x - 2x + 2$
 $= -8x^3 + 6x + 2$

Δ. $(x+\psi)^2 = x^2 + 2x\psi + \psi^2$

$(\psi-2)^2 = \psi^2 - 2 \cdot \psi \cdot 2 + 2^2 = \psi^2 - 4\psi + 4$

$(x+2\psi)^2 = x^2 + 2 \cdot x \cdot 2\psi + (2\psi)^2 = x^2 + 4x\psi + 4\psi^2$

$(4x-3\psi)^2 = (4x)^2 - 2 \cdot 4x \cdot 3\psi + (3\psi)^2 = 16x^2 - 24x\psi + 9\psi^2$

$(2x+1)(2x-1) = (2x)^2 - 1^2 = 4x^2 - 1$

$(2x+1)^3 = (2x)^3 + 3(2x)^2 \cdot 1 + 3 \cdot 2x \cdot 1^2 + 1 = 8x^3 + 3 \cdot 4x^2 + 6x + 1$
 $= 8x^3 + 12x^2 + 6x + 1$

$$\begin{aligned}
 (3x-2)^3 &= (3x)^3 - 3 \cdot (3x)^2 \cdot 2 + 3 \cdot 3x \cdot 2^2 - 2^3 \\
 &= 27x^3 - 3 \cdot 9x^2 \cdot 2 + 3 \cdot 3x \cdot 4 - 8 \\
 &= 27x^3 - 54x^2 + 36x - 8
 \end{aligned}$$

$$\begin{aligned}
 \text{Ε. } P &= 3x^2(x+3)^2 - (x+3)(x-3) - (x+2)^3 = \\
 &= 3x^2(x^2 + 2 \cdot x \cdot 3 + 3^2) - (x^2 - 3^2) - (x^3 + 3 \cdot x^2 \cdot 2 + 3x \cdot 2^2 + 2^3) \\
 &= 3x^2(x^2 + 6x + 9) - (x^2 - 9) - (x^3 + 6x^2 + 12x + 8) \\
 &= 3x^4 + 18x^3 + 27x^2 - x^2 + 9 - x^3 - 6x^2 - 12x - 8 \\
 &= 3x^4 + 17x^3 + 20x^2 - 12x + 1
 \end{aligned}$$

$$\text{ΣΤ. 1. } 2x^3 + 4x^2 = 2x^2(x+2) \quad [\text{κοινός παράγοντας}]$$

$$\begin{aligned}
 \text{2. } \underline{2x} + \underline{6\omega} + \underline{x^2} + \underline{3x\omega} &= [\text{ομαδοποίηση}] \\
 &= x(2+x) + 3\omega(2+x) = (2+x)(x+3\omega)
 \end{aligned}$$

$$\text{3. } x^2 - 6x + 9 = x^2 - 2 \cdot 3x + 3^2 = (x-3)^2 \quad [\text{ταυτότητα}]$$

$$\text{4. } 4x^2 - 9\psi^2 = (2x)^2 - (3\psi)^2 = (2x+3\psi)(2x-3\psi) \quad [\text{διαφορά τετραγώνων}]$$

$$\text{5. } x^2 + 5x + 6 = x^2 + 2x + 3x + 6x = (x+2)(x+3)$$

$$\text{6. } x^3 - 6x^2 + 9x = x(x^2 - 6x + 9) = x(x-3)^2$$

$$\begin{aligned}
 \text{7. } \underline{2x^5} - \underline{4x^4} + \underline{2x^3} - \underline{4x^2} &= 2x^3(x^2+1) - 4x^2(x^2+1) \quad [\text{ομαδοποίηση}] \\
 &= (x^2+1)(2x^3 - 4x^2) \\
 &= (x^2+1) \cdot 2x^2 \cdot (x-2) \\
 &= 2x^2(x^2+1)(x-2)
 \end{aligned}$$

$$\text{8. } (5x-10)(x^2-1) - (7x-14)(x-1)^2 = [\text{κοινός παράγοντας}]$$

$$\begin{aligned}
 8. &= (x^2-1) [(5x-10) - (7x-14)] = \\
 &= (x^2-1) (\underline{5x-10} - \underline{7x+14}) = (x^2-1) (-2x+4) = \\
 &= (x+1)(x-1) \cdot -2(x-2) = -2(x+1)(x-1)(x-2)
 \end{aligned}$$

$$2. \alpha. \quad A = \frac{(a-b)^2}{a^2-b^2} \cdot \frac{(a+b)}{a-b} = \frac{(a-b)^2}{(a+b)(a-b)} \cdot \frac{(a+b)}{(a-b)} =$$

$$A = \frac{\cancel{(a-b)}^2 \cdot (a+b)}{(a+b) \cdot \cancel{(a-b)}} = 1$$

$$B = \frac{x}{x-1} \cdot \frac{x^2-x}{x^3} = \frac{x(x^2-x)}{(x-1)x^3} = \frac{\cancel{x} \cdot \cancel{x} (x-1)}{(x-1) \cancel{x}^2} = \frac{1}{x}$$

$$\Gamma = \frac{2a-b}{a-b} \cdot \frac{a}{\beta-2a} \cdot \frac{(b-a)^2}{a^3} = \frac{(2a-b) a (b-a)^2}{(a-b) (\beta-2a) a^3} \quad \text{αλλαγή}$$

$$\Gamma = \frac{\cancel{a} (2a-b) (a-b)^2}{(a-b) \cdot (\beta-2a) \cancel{a}^2} = \frac{-(\beta-2a)(a-b)}{(\beta-2a) a^2} = -\frac{a-b}{a^2}$$

$$\Delta = (a-2a^2) \cdot \frac{1}{a} \cdot \left(-\frac{1}{a+2}\right)^2 = a(1-2a) \cdot \frac{1}{a} \left[+\frac{1}{(a-2)^2}\right] =$$

$$\Delta = a(1-2a) \frac{1}{a} \frac{1}{(a-2)^2} = \frac{a(1-2a)}{a(a-2)^2} = \frac{1-2a}{(a-2)^2}$$

$$\beta) A = \frac{x^3}{x^2-1} : \frac{x^3+2x^2+x}{2x^2} = (\text{κάνω παραγοντοποίηση})$$

$$A = \frac{x^3}{(x+1)(x-1)} : \frac{x(x^2+2x+1)}{2x^2} =$$

$$A = \frac{x^3}{(x+1)(x-1)} \cdot \frac{2x^2}{x(x^2+2x+1)} \quad (\text{αυτι για διαίρεση, ηρ/ηρ})$$

$$A = \frac{x^3 \cdot 2x^{\cancel{2}}}{(x+1)(x-1) \cdot \cancel{x} (x^2+2x+1)} = \frac{2x^4}{(x+1)(x-1)(x+1)^2}$$

$$A = \frac{2x^4}{(x+1)^3(x-1)}$$

$$B = \frac{1-9x^2}{8x} : \frac{9x^2-6x+1}{12x^2-4x} = (\text{παραγοντοποίηση})$$

$$B = \frac{1-(3x)^2}{8x} : \frac{(3x)^2-6x+1}{4x(3x-1)} =$$

$$B = \frac{(1+3x)(1-3x)}{8x} : \frac{(3x-1)^2}{4x(3x-1)} =$$

$$B = \frac{(1+3x)(1-3x)}{8x} \cdot \frac{4x \cdot \cancel{(3x-1)}}{\cancel{(3x-1)}^2} = (\text{απλοποίηση})$$

$$B = \frac{\cancel{4x} (1+3x)(1-3x)}{2 \cdot \cancel{8x} (3x-1)}$$

$$B = \frac{-(1+3x)(3x-1)}{2(3x-1)}$$

$$B = \frac{-(1+3x)}{2}$$

$$\Gamma = \frac{\frac{x+2}{x-2}}{\frac{x-2}{x+2}} : \frac{x^2+4x+4}{x^2-4x+4} \quad (\text{συνθετο κλάσμα σε απλό})$$

$$\Gamma = \frac{(x+2)(x+2)}{(x-2)(x-2)} : \frac{(x+2)^2}{(x-2)^2} \quad (\text{παράγοντισμός})$$

$$\Gamma = \frac{\cancel{(x+2)}\cancel{(x+2)}\cancel{(x+2)}^2}{\cancel{(x-2)}\cancel{(x-2)}\cancel{(x+2)}^2} = 1$$

3. 1. $x^2 - 3x = 0 \Rightarrow x(x-3) = 0 \Rightarrow \boxed{x=0}$ ή $x-3=0$
 $\boxed{x=3}$

2. $x^2 - 16 = 0 \Rightarrow x^2 - 4^2 = 0 \Rightarrow (x+4)(x-4) = 0 \Rightarrow$
 $x+4=0$ ή $x-4=0$
 $\boxed{x=-4}$ ή $\boxed{x=4}$

3. $x^2 + 9 = 0$ αδύνατη ως άθροισμα θετικών όρων

4. $4x^2 - 1 = 0 \Rightarrow (2x)^2 - 1 = 0 \Rightarrow (2x+1)(2x-1) = 0 \Rightarrow$
 $\Rightarrow 2x+1=0$ ή $2x-1=0$
 $\Rightarrow 2x=-1$ ή $2x=1$
 $\Rightarrow \boxed{x=-1/2}$ ή $\boxed{x=1/2}$

$$5. \quad x^3 + 4x = 0 \Rightarrow x(x^2 + 4) = 0 \Rightarrow \boxed{x=0} \quad \text{u} \quad x^2 + 4 = 0$$

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$$6. \quad 2x^3 - 4x^2 = 0 \Rightarrow 2x^2(x-4) = 0 \Rightarrow 2x^2 = 0 \quad \text{u} \quad x-4 = 0$$

$\boxed{x=0}$ $\text{u} \quad \boxed{x=4}$

$$7. \quad x^2 - 7x + 6 = 0 \Rightarrow \underbrace{x^2 - x - 6x + 6 = 0} \Rightarrow \underbrace{x(x-1) - 6(x-1) = 0} \Rightarrow$$

$$\Rightarrow (x-1)(x-6) = 0 \Rightarrow x-1 = 0 \quad \text{u} \quad x-6 = 0$$

$\boxed{x=1}$ $\text{u} \quad \boxed{x=6}$

$$8. \quad x^2 - x - 2 = 0 \Rightarrow \underbrace{x^2 - 2x + x - 2 = 0} \Rightarrow x(x-2) + (x-2) = 0 \Rightarrow$$

$$\Rightarrow (x-2)(x+1) = 0 \Rightarrow x-2 = 0 \quad \text{u} \quad x+1 = 0$$

$\boxed{x=2}$ $\text{u} \quad \boxed{x=-1}$

$$9. \quad x^2 - 6x + 5 = 0 \Rightarrow \underbrace{x^2 - 5x - x + 5 = 0} \Rightarrow \underbrace{x(x-5) - (x-5) = 0} \Rightarrow$$

$$\Rightarrow (x-5)(x-1) = 0 \Rightarrow x-5 = 0 \quad \text{u} \quad x-1 = 0$$

$\boxed{x=5}$ $\text{u} \quad \boxed{x=1}$

$$10. \quad 2x^2 - 3x - 2 = 0 \Rightarrow a = 2 \quad \beta = -3 \quad \gamma = -2$$

$$\Delta = \beta^2 - 4a\gamma = (-3)^2 - 4 \cdot 2 \cdot (-2) = 9 + 16 = 25 > 0$$

$$x_{1/2} = \frac{-\beta \pm \sqrt{\Delta}}{2a} = \frac{-(-3) \pm \sqrt{25}}{2 \cdot 2} = \frac{3 \pm 5}{4} = \begin{cases} x_1 = \frac{3+5}{4} = \frac{8}{4} = \boxed{2} \\ x_2 = \frac{3-5}{4} = \frac{-2}{4} = \boxed{-\frac{1}{2}} \end{cases}$$

$$11. \quad x^2 - 6x + 9 = 0 \Rightarrow x^2 - 6x + 3^2 = 0 \Rightarrow (x-3)^2 = 0 \Rightarrow x-3 = 0 \Rightarrow \boxed{x=3}$$

$$12. \quad x^2 - 6x + 10 = 0$$

$$a = 1$$

$$\beta = -6$$

$$\gamma = 10$$

$$\Delta = \beta^2 - 4a\gamma = (-6)^2 - 4 \cdot 1 \cdot 10 = 36 - 40 = -4 < 0$$

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$$\begin{aligned}
 13. \quad (x^2-4)(x^2-1) &= 0 \Rightarrow x^2-4=0 \quad \vee \quad x^2-1=0 \rightarrow \\
 &\Rightarrow (x+2)(x-2)=0 \quad \vee \quad (x+1)(x-1)=0 \\
 &\Rightarrow x+2=0 \quad \vee \quad x-2=0 \quad \vee \quad x+1=0 \quad \vee \quad x-1=0 \\
 &\Rightarrow \boxed{x=-2} \quad \vee \quad \boxed{x=2} \quad \vee \quad \boxed{x=-1} \quad \vee \quad \boxed{x=1}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad x(x-3) - 4(x-3) &= 0 \Rightarrow (x-3)(x-4) = 0 \Rightarrow \\
 &\Rightarrow \boxed{x=3} \quad \vee \quad \boxed{x=4}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad x(x+2) - (x+2)^2 + 3x(x-1) + 6x &= 0 \Rightarrow \\
 x^2 + 2x - (x^2 + 4x + 4) + 3x^2 - 3x + 6x &= 0 \Rightarrow \\
 \underline{x^2} + \underline{2x} - \underline{x^2} - \underline{4x} - \underline{4} + \underline{3x^2} - \underline{3x} + \underline{6x} &= 0 \Rightarrow \\
 x^2 + x^2 + 3x^2 - 3x + 6x - 4 &= 0 \Rightarrow
 \end{aligned}$$

$$5x^2 + 3x - 4 = 0$$

$$a = 5$$

$$b = 3$$

$$c = -4$$

$$\Delta = b^2 - 4ac = 3^2 - 4 \cdot 5 \cdot (-4) = 9 + 80 = 89$$

$$\begin{aligned}
 x_{1,2} &= \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-3 \pm \sqrt{89}}{2 \cdot 5} = \frac{-3 \pm \sqrt{89}}{10} \\
 x_1 &= \frac{-3 + \sqrt{89}}{10} = \frac{8}{10} = \boxed{\frac{4}{5}} \\
 x_2 &= \frac{-3 - \sqrt{89}}{10} = \frac{-10}{10} = \boxed{-1}
 \end{aligned}$$

$$4) \quad A. \quad MN = \frac{B\Gamma}{2}$$

$$\frac{BC}{CZ} = \frac{BZ}{AZ}$$

$$B, \quad 1, \rightarrow \Sigma$$

$$2, \rightarrow \Lambda$$

$$3, \rightarrow \Sigma$$

$$4, \rightarrow \Lambda$$

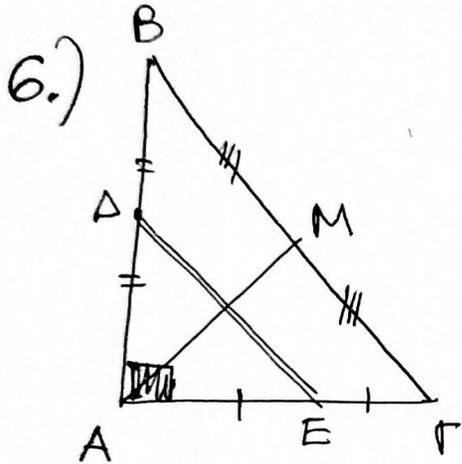
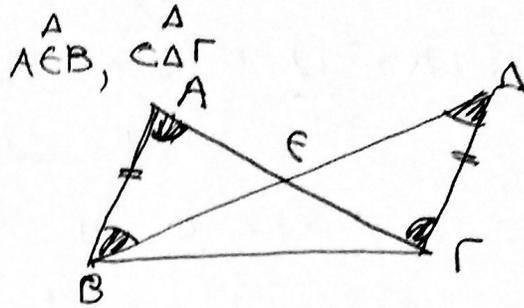
5.) Συγκρίνω τα τρίγωνα

$$AB = \Gamma\Delta \text{ (Δεδ.)}$$

$$\hat{A} = \hat{\Gamma} \text{ (ως ενός εναντίου)}$$

$$\hat{B} = \hat{\Delta} \text{ (" ")}$$

$$\text{ισα} \Rightarrow \boxed{AE = \epsilon\Gamma}$$



6.) α) Η ΑΜ είναι διάμεσος ορθογωνίου τριγώνου και άρα θα είναι ίση με το μισό της υποκείμενης.

$$\text{Άρα } AM = \frac{B\Gamma}{2} \Rightarrow AM = BM = M\Gamma \text{ (I)}$$

Αυτό σημαίνει ότι τα τρίγωνα $\triangle ABM$ και $\triangle AM\Gamma$ είναι ισοσκελή.

Επίσης η ΔΕ συνδέει τα μέσα δύο πλευρών ενός τριγώνου και σύμφωνα με τη θεωρία θα είναι ίση με $\Delta\epsilon = \frac{B\Gamma}{2}$ (II) το μισό της τρίτης πλευράς

Απο (I) και (II) σχέση έχουμε ότι $\boxed{\Delta\epsilon = AM}$