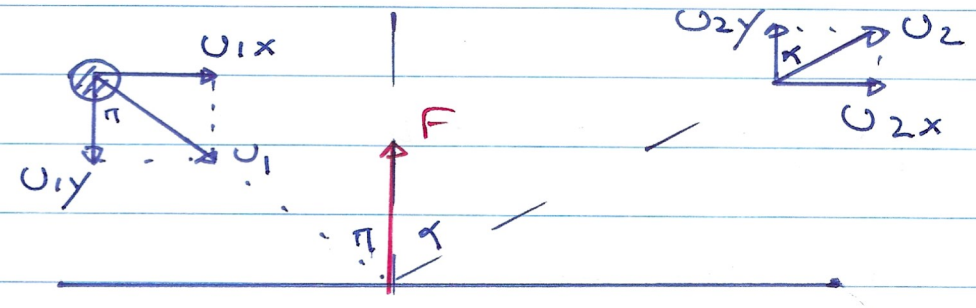
ΘΕΜΑ Α

$A_1 \to \gamma$ $A_2 \to B$ $A_3 \to B$ $A_4 \to \delta$ $A_5 \to \Sigma$

(A2)



Ανελαστική, υπάρχει Εαπ., άρα $K_2 < K_1 \to \dots$

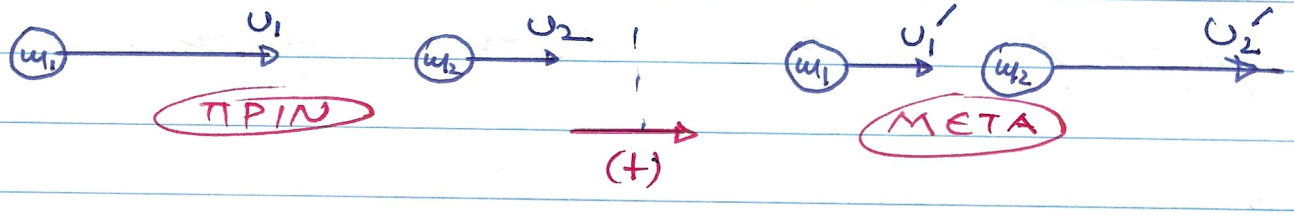
$\dots \to \boxed{U_2 < U_1}$

$\vec{P}_{\text{ΠΡΙΝ}} = \vec{P}_{\text{ΜΕΤΑ}} \to m U_{1x} = m U_{2x} \to$

$U_1 \cdot \text{συν}\pi = U_2 \cdot \text{συν}\alpha \to \frac{\text{συν}\alpha}{\text{συν}\pi} = \frac{U_1}{U_2} > 1 \to \text{συν}\alpha > \text{συν}\pi \to \boxed{\hat{\alpha} > \hat{\pi}}$

ΘΕΜΑ Β

Β1



$\frac{\upsilon_1}{\upsilon_2} = 4$ $\qquad$ $m_1 \underset{j}{=} f(m_2)$ $\qquad$ $\frac{\upsilon_1'}{\upsilon_2'} = \frac{1}{4}$

ΛΥΣΗ:

$$\upsilon_1' = \frac{m_1 - m_2}{m_1 + m_2}\upsilon_1 + \frac{2m_2}{m_1 + m_2}\upsilon_2$$

$$\upsilon_2' = \frac{2m_1}{m_1 + m_2}\upsilon_1 + \frac{m_2 - m_1}{m_1 + m_2}\upsilon_2$$

$$\rightarrow \frac{1}{4} = \frac{(m_1 - m_2)4\upsilon_2 + 2m_2\upsilon_2}{2m_1 \cdot 4\upsilon_2 + (m_2 - m_1)\upsilon_2} \rightarrow$$

$$\frac{1}{4} = \frac{4m_1 - 4m_2 + 2m_2}{2m_1 \cdot 4 + m_2 - m_1} \rightarrow \frac{1}{4} = \frac{4m_1 - 2m_2}{7m_1 + m_2} \rightarrow$$

$$16m_1 - 8m_2 = 7m_1 + m_2 \rightarrow 9m_1 = 9m_2 \rightarrow \boxed{m_1 = m_2}\ (*)$$

(α)
2Δ + 6Δ

Ή:

$$\upsilon_1 + \upsilon_1' = \upsilon_2 + \upsilon_2' \rightarrow$$

$$4\upsilon_2 + \upsilon_1' = \upsilon_2 + 4\upsilon_1' \rightarrow$$

$$3\upsilon_2 = 3\upsilon_1' \rightarrow$$

$$\boxed{\upsilon_2 = \upsilon_1'} \xrightarrow[\text{ΤΑΧΥΤΗΤΩΝ}]{\text{ΑΝΤΙΣΤΡΟΦΗ}} \boxed{m_1 = m_2}$$

Β2

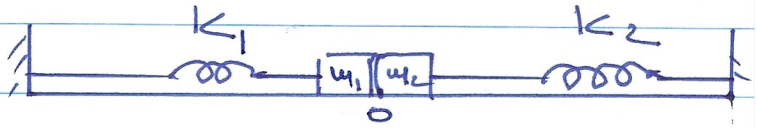
$$F_{ΕΠ} = \frac{F_{ΕΠ.max}}{4} \rightarrow -Dx = \frac{-DA}{4} \rightarrow \boxed{x = \frac{A}{4}}$$

$$\frac{K}{U} = \frac{E_T - U}{U} = \frac{\frac{1}{2}DA^2 - \frac{1}{2}D\frac{A^2}{16}}{\frac{1}{2}D\frac{A^2}{16}} =$$

$$= \frac{15/16}{1/16} \rightarrow \boxed{\frac{K}{U} = 15}$$

(Β) 2μ + 6μ

Β3 Για την ταλάντωση του m_1:

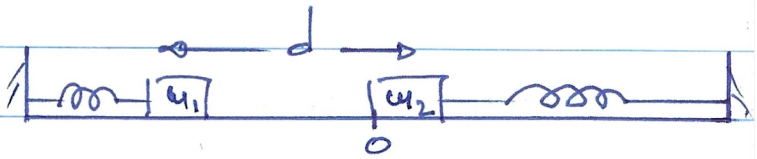


$A_1 = D = K_1 = K$

$K = m_1\omega_1^2 \rightarrow \omega_1 = \sqrt{\frac{K}{m}}$

$U_{max.1} = \omega_1 \cdot A_1 \rightarrow$

$U_{max.1} = \sqrt{\frac{K}{m}} \cdot d$ (1)



Κεντρική πλαστική

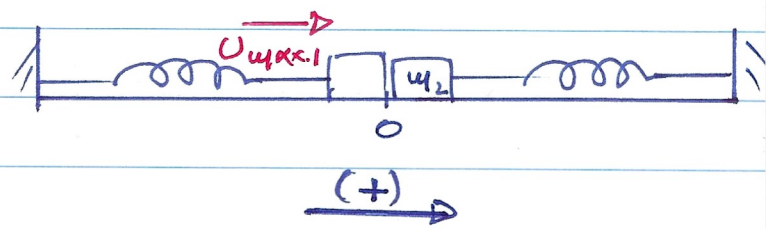
Κρούση στο (Ο):

$\Sigma F_{εξ} = 0 \xrightarrow{Α.Δ.Ο.} \vec{P}_{ΠΡΙΝ} = \vec{P}_{ΜΕΤΑ} \rightarrow$

$mU_{max.1} = (m+m)U_{max.2} \rightarrow$

$U_{max.1} = 2 \cdot U_{max.2} \rightarrow$

$\omega_1 \cdot A_1 = 2 \cdot \omega_2 A_2 \rightarrow$



$$\sqrt{\frac{K}{m}} A_1 = 2\sqrt{\frac{2K}{2m}} \cdot A_2 \rightarrow \boxed{A_1 = 2A_2}$$

(Β) 2μ + 7μ

ΘΕΜΑ Γ

$m = 1\,kg$, $K = 2 - 50x^2$, $t_0 = 0 \rightarrow +A$

ΛΥΣΗ:

Γ1

$\Delta x =$;

$t_{min} =$;

$x = A \cdot \eta\mu(\omega t + \phi_0)$ $\underset{\rightarrow}{t_0 = 0}$ $+A = A \cdot \eta\mu\phi_0 \rightarrow$

$\eta\mu\phi_0 = 1 \rightarrow$ $\boxed{\phi_0 = \frac{\pi}{2}\,rad}$

Α.Δ.Ε.Τ $\rightarrow$ $E_T = K + U \rightarrow K = E_T - U$

Όπως: $K = 2 - 50x^2$

$\boxed{E_T = 2\,J}$ και $U = 50x^2 = \frac{1}{2}Dx^2 \rightarrow \boxed{D = 100\,N/m}$

Άρα $E_T = \frac{1}{2}DA^2 = 2 \rightarrow A^2 = \frac{4}{D} = \frac{4}{100} \rightarrow \boxed{A = 0{,}2\,m}$

$D = m\omega^2 \rightarrow \omega = \sqrt{\frac{D}{m}} = \sqrt{\frac{100}{1}} \rightarrow \boxed{\omega = 10\,rad/s}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{10} \rightarrow \boxed{T = \frac{\pi}{5}\,s}$

$\Delta x = 2A = 2 \cdot 0{,}2 \rightarrow \boxed{\Delta x = 0{,}4\,m}$, $\boxed{t_{min} = \frac{T}{2} = \frac{\pi}{10}\,s}$

(3+) * (3+) *

Γ2

$\Sigma F =$;

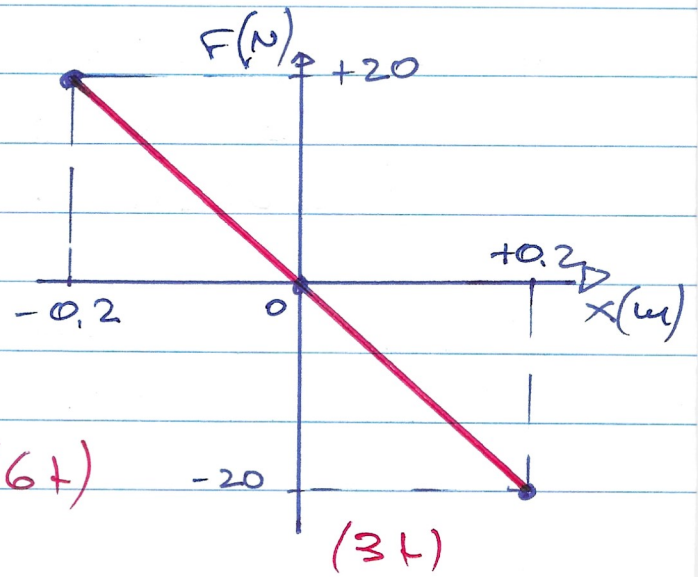
$\Sigma F = f(x)$

$\Sigma F = -Dx \rightarrow$

$\boxed{\Sigma F = -100x}$

$\boxed{-0{,}2\,m \le x \le +0{,}2\,m}$

(3+)



(6+) (3+)

(Γ3)

$\Delta x = ;$

$S_{ολ} = ;$

$t_0 = 0 \rightarrow t_1 = \frac{2\pi}{5} s$

$x = A\eta\mu(\omega t + \phi_0) =$

$= 0{,}2\eta\mu\left(10\,\frac{2\pi}{5} + \frac{\pi}{2}\right) =$

$= 0{,}2\cdot\eta\mu\left(4\pi + \frac{\pi}{2}\right) =$

$= 0{,}2\eta\mu\frac{\pi}{2} \rightarrow \boxed{x = 0{,}2\,\mu}$

ή: $t_1 = \frac{2\pi}{5} s = 2\,\frac{\pi}{5} = 2T \rightarrow$

έχει εκτελέσει 2 πλήρεις ταλαντώσεις και επέστρεψε στο $+A$.

Άρα $\Delta x = x_{τελ} - x_{αρχ} = 0{,}2 - 0{,}2 \rightarrow \boxed{\Delta x = 0}$ (3μ) *

$S_{ολ} = (A + A + A + A)2 = 8A = 8\cdot 0{,}2 \rightarrow \boxed{S_{ολ} = 1{,}6\,\mu}$ (3μ)

(Γ4)

$t_2 = \frac{T}{12}$

$\frac{K}{E_T} = ;(\%)$

$\upsilon = \upsilon_{μεγ}\sigma\upsilon\nu(\omega t + \phi_0) =$

$= 10\cdot 0{,}2\,\sigma\upsilon\nu\left(10\,\frac{T}{12} + \frac{\pi}{2}\right) =$

$= 2\sigma\upsilon\nu\left(\frac{\cancel{5}}{6}\cdot\frac{\pi}{\cancel{5}} + \frac{\pi}{2}\right) =$

$= 2\sigma\upsilon\nu\left(\frac{1}{6}\pi + \frac{3}{2}\pi\right) = 2\sigma\upsilon\nu\,\frac{4\pi}{6} = 2\sigma\upsilon\nu\,\frac{2\pi}{3} =$

$= 2\left(-\frac{1}{2}\right) \rightarrow \boxed{\upsilon = -1\,m/s}$ (3μ)

$K = \frac{1}{2}m\upsilon^2 = \frac{1}{2}\cdot 1\cdot 1^2 \rightarrow \boxed{K = 0{,}5\,J}$ (2μ)

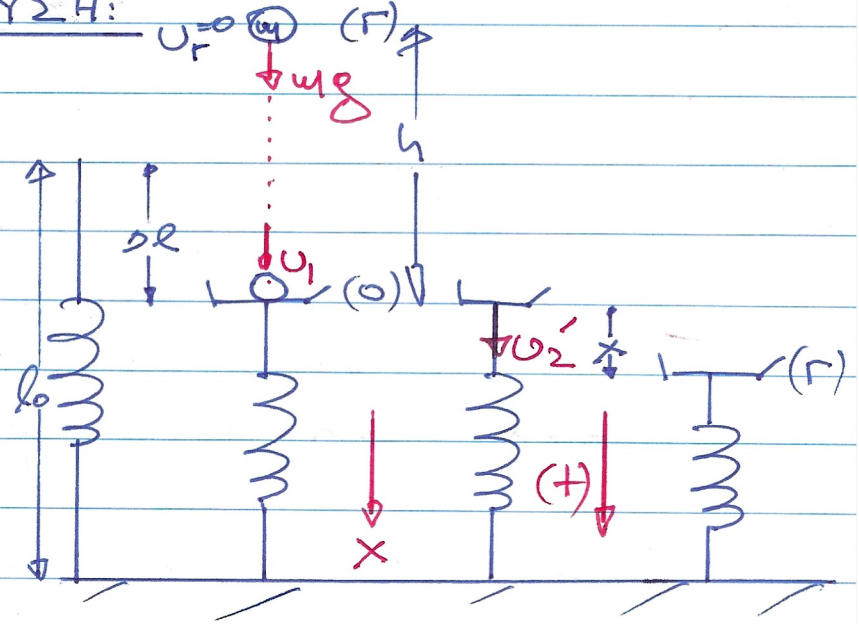
$\frac{K}{E_T} = \frac{1/2}{2} = \frac{1}{4} \rightarrow K = \frac{1}{4}E_T\cdot 100\% \rightarrow$

$\boxed{K = 25\%\,E_T}$ (2μ)

ΘΕΜΑ Δ

$K = 1000\,N/m$, $M = 10\,kg$, $h = 0{,}8\,m$, $m = 6\,kg$

ΛΥΣΗ:



Δ1

$U_1' = ;$

$U_2' = ;$

Ισορροπία της M στην Ο

$\Sigma F_x = 0 \rightarrow$

$F_{\epsilon\lambda} = Mg = K\cdot\Delta\ell \rightarrow$

$\Delta\ell = \frac{Mg}{K} = \frac{10\cdot 10}{1000} \rightarrow$

$$\boxed{\Delta\ell = 0{,}1\,m}$$

Κίνηση της m, Γ → Ο: Θ.Ε.Ε. → $\frac{1}{2}mU_1^2 - 0 = mgh \rightarrow U_1 = \sqrt{2gh} = \sqrt{2\cdot 10\cdot \frac{8}{10}} \rightarrow$

$$\boxed{U_1 = 4\,m/s}$$

Μετωπική Ελαστική Κρούση των $M-m$ στην Ο

$U_1' = \frac{m-M}{m+M}U_1 = \frac{6-10}{6+10}\cdot 4 \rightarrow$

$$\boxed{U_1' = -1{,}6\,m/s}$$

(2μ)

$U_2' = \frac{2m}{m+M}U_1 = \frac{2\cdot 6\cdot 4}{6+16} \rightarrow$

$$\boxed{U_2' = 3\,m/s = U_{max}}$$

(2μ)

Δ2

Α.Α.Τ.;

$f = ;$

Έκτροπή δίσκου στην (Γ)

$\Sigma F_x = F_{\epsilon\lambda}^{(r)} - Mg = K(\Delta\ell + x) - Mg$ → (2μ)

$\Sigma F_x = -Kx$ Α.Α.Τ. $\rightarrow D = K = 1000\,N/m$

$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{1000}{10}} \rightarrow$

$$\boxed{\omega = 10\,rad/s}$$

$T = \frac{2\pi}{\omega} \rightarrow$

$$\boxed{T = \frac{\pi}{5}\,s}$$

$f = \frac{1}{T} \rightarrow$

$$\boxed{f = \frac{5}{\pi}\,Hz}$$

(2μ)

Επειδή τη $t_0 = 0$ { $\to x = 0$ δεν υπάρχει ϕ_0 / $\to U > 0$

Δ3

$P = f(t)$

$t_0 = 0 \to 2T$

$U_\Sigma = U_{max} = \omega A \to A = \frac{U_{max}}{\omega} = \frac{3}{10}$

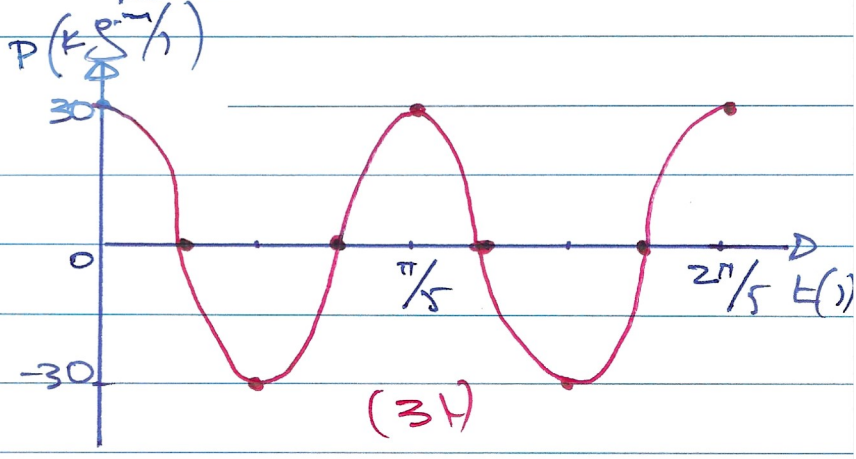
$\to \boxed{A = 0,3\,m}$

$P = MU =$

$= M \cdot U_{max} \sigma\upsilon\nu\omega t =$

$= 10 \cdot 3 \sigma\upsilon\nu 10t \to$

$\boxed{P = 30 \sigma\upsilon\nu 10t \quad 0 \le t \le 2T = \frac{2\pi}{5}}$ (2Μ)



(3Μ)

Δ4

2η φορά στο l_0

$\frac{dK}{dt} = ;$

Όταν ο δίσκος Μ διέρχεται από τη Θ.Φ.Μ. του ελατηρίου έχει $\boxed{x_1 = -\Delta l = -0,1\,m}$

Α.Δ.Ε.Τ. $\to \frac{1}{2} D A^2 = \frac{1}{2} M U_1^2 + \frac{1}{2} D x_1^2 \to$ (1Μ)

$U_1^2 = \frac{D(A^2 - x_1^2)}{M} = \frac{1000(0,3^2 - 0,1^2)}{10} = 8 \to \boxed{U_1 = +2\sqrt{2}\,m/s}$ (2Μ)

$\frac{dK}{dt} = \frac{dW_{\Sigma F}}{dt} = \frac{\Sigma F \cdot dx}{dt} = -Dx_1 \cdot U_1 = -1000 \cdot (-0,1) \cdot 2\sqrt{2} \to$

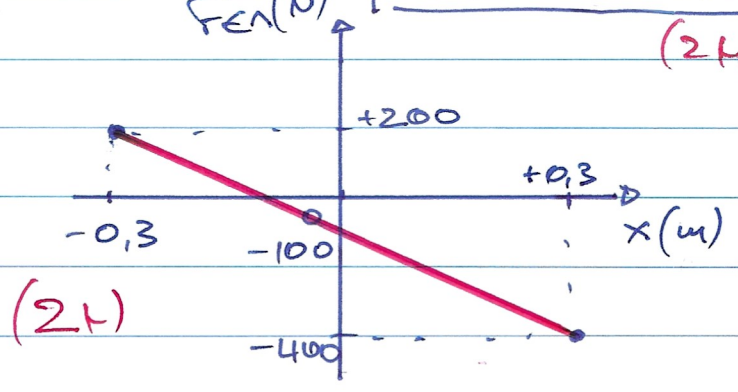
$\boxed{\frac{dK}{dt} = +200\sqrt{2}\,J/s}$ (2Μ)

Δ5

$\vec{\Sigma F} = -D\vec{x} \to \vec{F}_{\epsilon\lambda} + \vec{w} = -D\vec{x} \to$

$F_{\epsilon\lambda} + w = -Dx \to$

$F_{\epsilon\lambda} = -w - Dx \to \boxed{F_{\epsilon\lambda} = -100 - 1000x \quad -0,3\,m \le x \le +0,3\,m}$ (2Μ)



(2Μ)

Δ6

$\frac{E_T}{U_{\epsilon\lambda}^{max}} = ;$

$\frac{E_T}{U_{\epsilon\lambda}^{max}} = \frac{\frac{1}{2} D \cdot A^2}{\frac{1}{2} k (\Delta l + A)^2} =$

$= \frac{A^2}{(\Delta l + A)^2} =$

$= \frac{(3/10)^2}{(1/10 + 3/10)^2} \to$

$\boxed{\frac{E_T}{U_{\epsilon\lambda}^{max}} = \frac{9}{16}}$ (3Μ)